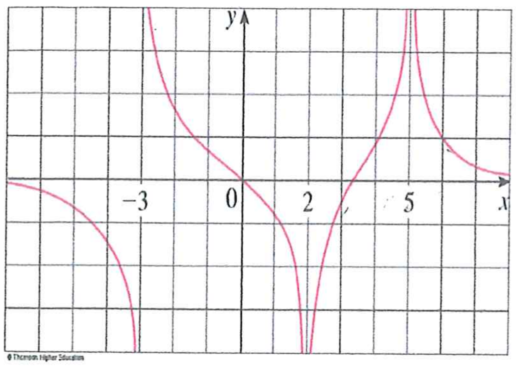
MAC 2311 Calculus I **PRACTICE TEST #1** Name Solutions

Chapter 2 Limits, Continuity, and Derivatives



1. For the function f whose graph is shown, find the following:

a. $\lim_{x\to5} f(x)$ ∞

b. $\lim_{x\to2} f(x)$ $-\infty$

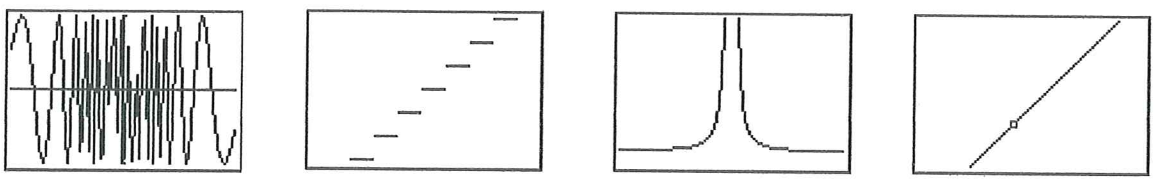
c. $\lim_{x\to6} f(x)$ 1

d. $\lim_{x\to0^+} f(x)$ 0

e. $\lim_{x\to\infty} f(x)$ 0

f. $\lim_{x\to-3} f(x)$ $\pm\infty$ OR DNE

2. Identify the following types of discontinuities from the four we discussed.



a. oscillation b. jump c. infinite d. removable

Find the limits of the following functions (do not use the table method for either problem).:

3. $\lim_{x\to0} \frac{\sqrt{x+4}-2}{x}$

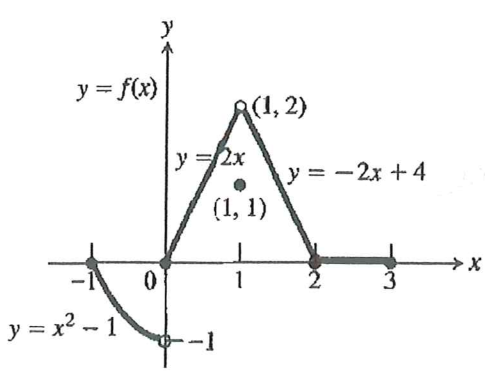
$$\lim_{x\to0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$$

$$\lim_{x\to0} \frac{x+4-4}{x(\sqrt{x+4}+2)} = \frac{\cancel{x}}{\cancel{x}(\sqrt{x+4}+2)}$$

$$\lim_{x\to0} \frac{1}{\sqrt{x+4}+2} = \boxed{\frac{1}{4}}$$

4. $\lim_{x\to5} \frac{x^2-6x+5}{x-5} = \frac{\cancel{(x-5)}(x-1)}{\cancel{x-5}}$

$$\lim_{x\to5} (x-1) = 5-1 = \boxed{4}$$



5. Use the graph at the left to answer the following:

a. $\lim_{x\to0^-} f(x) =$ -1

b. $\lim_{x\to0^+} f(x) =$ 0

c. $\lim_{x\to0} f(x) =$ DNE

d. $\lim_{x\to1} f(x) =$ 2

e. Prove that f is continuous at x = 2. (there are 3 parts to this proof)

1.) limit exists $\lim_{x\to2} f(x) = 0$

2.) $f(2)$ exists $f(2) = 0$

3.) $\lim_{x\to2} f(x) = f(2)$ $0 = 0$

6. Let $f(x) = \frac{x^2+1}{2x^2-3x-2}$.

a. What is $\lim_{x\to\infty} f(x)$? $\underline{\frac{1}{2}}$

$$\lim_{x\to\infty} \frac{\frac{x^2+1}{x^2}}{\frac{2x^2-3x-2}{x^2}} = \frac{1+\cancel{\frac{1}{x^2}}^{\,0}}{2+\cancel{\frac{3}{x}}^{\,0}-\cancel{\frac{2}{x^2}}^{\,0}} = \frac{1}{2}$$

b. Is $\lim_{x\to-\infty} f(x)$ the same as $\lim_{x\to\infty} f(x)$? Why or why not?

Yes. As $x \to -\infty$ $f(x) \to \frac{1}{2}$

As $x \to +\infty$ $f(x) \to \frac{1}{2}$

Neg. values of x as $x \to -\infty$ are squared making them positive.

c. Determine any vertical asymptotes.

$2x^2 - 3x - 2 = 0$

$(2x+1)(x-2) = 0$

$2x+1=0 \quad x-2=0$

$$\boxed{x = -\tfrac{1}{2} \qquad x = 2}$$

d. Explain how to find the horizontal asymptotes using **calculus**.

Divide both numerator and denominator by the highest degree of x in the denominator. After simplifying take the limit of the function as $x \to \infty$.

7. Find the equation of the tangent line to the function $f(x) = x^2 - 2x + 4$ at the point (3, 7). Use the definition of a derivative to find the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left((x+h)^2 - 2(x+h)+4\right) - \left(x^2-2x+4\right)}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h + \cancel{4} - \cancel{x^2} + \cancel{2x} - \cancel{4}}{h} = \lim_{h\to 0} \frac{2xh+h^2-2h}{h} = \lim_{h\to 0} 2x+h-2$$

$f'(x) = 2x - 2$

$f'(3) = 2(3) - 2$

$f'(3) = 4$ (slope of tangent)

$y - 7 = 4(x-3)$

$y - 7 = 4x - 12$

$\boxed{y = 4x - 5}$ equation of line tangent to $f(x) = x^2 - 2x + 4$ at the point (3,7).

8. Use the $\varepsilon-\delta$ definition of a limit to prove that the $\lim_{x\to 2}(2x-3)=1$. Include a graph that illustrates your proof.

STEP 1: Given that $\varepsilon > 0$, we need a $\delta > 0$, such that if $0<|x-2|<\delta$ then

$|(2x-3)-1|<\varepsilon$

$|2x-4|<\varepsilon$

$|2(x-2)|<\varepsilon$

$2|x-2|<\varepsilon$

$|x-2|<\varepsilon/2$

$\boxed{\text{Thus } \delta = \varepsilon/2}$

STEP 2: Given that $\varepsilon > 0$, we choose for $\delta = \varepsilon/2$, then if $0<|x-2|<\delta$ then, $|(2x-3)-1|<\varepsilon$. by the Precise Definition of a limit.

(See graph at the end of this document)

9. Use the definition of derivative to find $f'(x)$ if $f(x)=mx+b$.

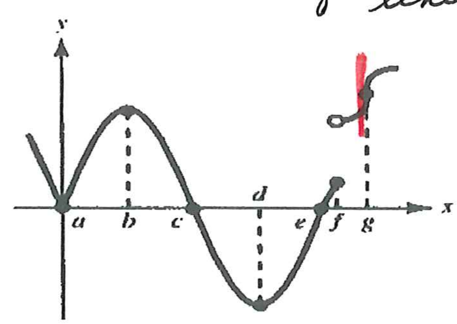
$$f'(x) = \lim_{h\to 0} \frac{(m(x+h)+b)-(mx+b)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{mx}+mh+\cancel{b}-\cancel{mx}-\cancel{b}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{m\cancel{h}}{\cancel{h}} \qquad \boxed{f'(x)=m}$$

Since $f(x)=mx+b$ is a linear function this means the slope of the line is the derivative of the line.

10. Use the graph at right to determine all x-values at which the function has no derivative. You may assume that what appears to be true in the graph is true. (You are looking at the points labeled a, b, c, etc…). Explain your reason(s) for your answer(s).

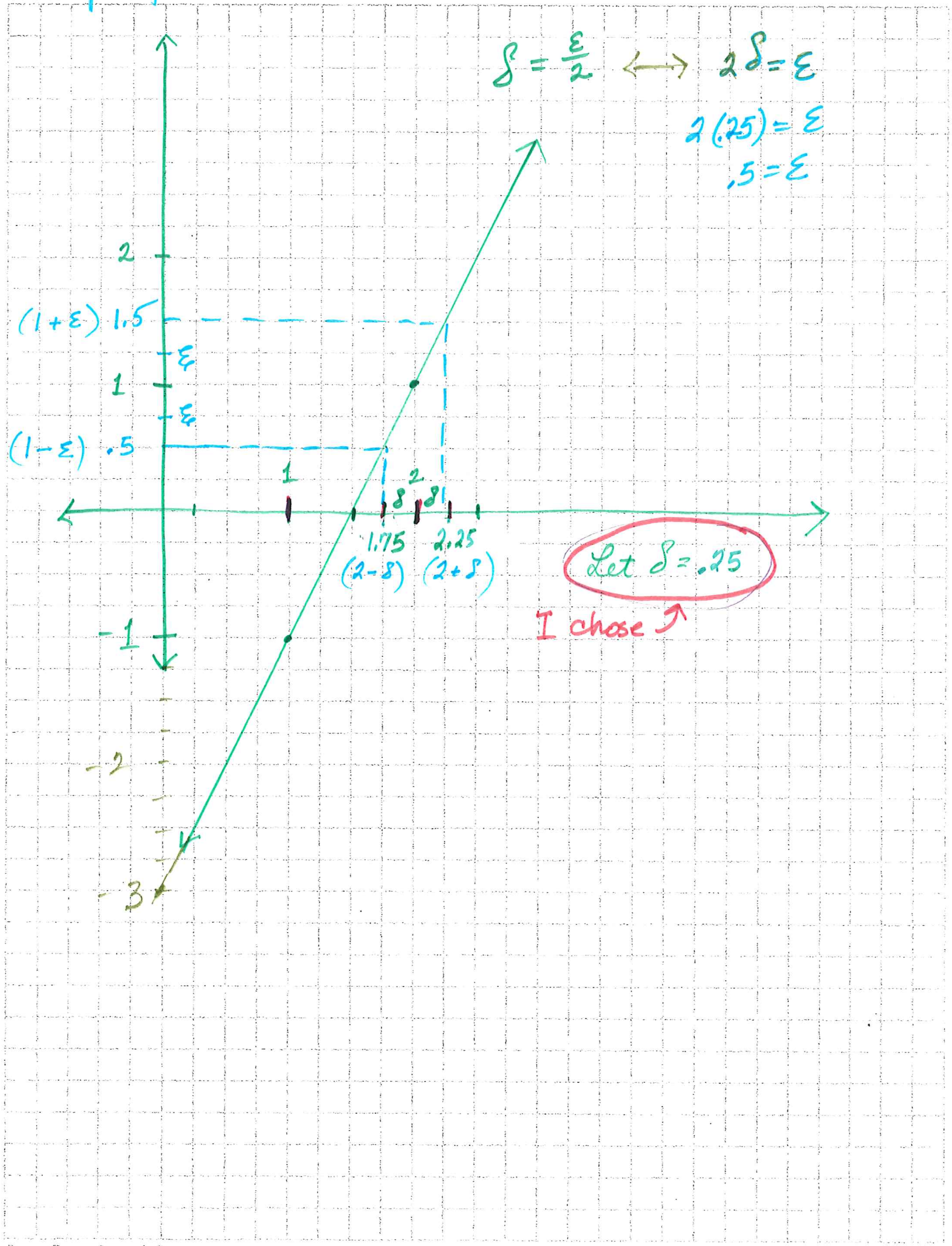


a – sharp points or corners

f – discontinuous

g – Vertical tangent

Graph for #8



Mrs. Robertson